Delamination from surface cracks in composite materials

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Some aspects of splitting and delamination of composite materials from surface flaws are discussed. The system modelled is one of an elastically homogeneous material containing aligned interfaces. This simplified model, although missing some of the details that would be associated with elastic inhomogeneity, does permit a discussion of some of the factors that govern this type of delamination.

1. Introduction

The strength of a number of technologically important systems is dependent upon the presence of internal interfaces. For example, planes of weakness may cause toughening in composite materials by deflecting and stopping cracks (Fig. 1a) [1]. A related problem is involved in the technology of electronic packaging; the interaction of cracks with the interface between a substrate and thin film determines whether decohesion occurs (Fig. 1b) [2]. In general, interfaces may separate materials with different elastic properties; the consequent elastic mismatch effects may then be very important [3–5]. However, the effect of an interface in an elastically homogeneous system is, in itself, very interesting and not completely understood.

The mechanism of delamination most often considered for composites is the one in which differing elastic properties between adjacent plies induces interlaminar shear and normal stresses near the boundaries of a specimen. These stresses cause separation of the interface, and delamination proceeds as an interlaminar crack that propagates from the edges of the specimens [6–9]. This type of delamination is completely dependent upon the differing elastic properties on either side of the interface. It would not occur in a homogeneous system.

There is another type of delamination which is induced by discontinuities in the geometry of the sample [10–17] and can occur in homogeneous systems. Two particular geometries which appear to be of practical importance are illustrated in Fig. 2. Delamination can initiate from the tip of a crack that exists on the edge (Fig. 2a) [11] or in the middle of a plane (Fig. 2b) [12–14]. This paper presents some results from fracture mechanics that are applicable to this problem. These results are then used to discuss the factors that govern delamination. It is emphasized that the details of the problem could be qualitatively changed if the materials on either side of the interface are different [5, 18–20]. Even when isotropic elasticity is assumed for both materials, the problem is considerably complicated. In particular, a modulus mismatch changes the nature of the stress singularity at the crack tip both for interface cracks [18–20] and for cracks terminating at an interface [20]. These singularities may have complex components; the resulting stress and displacement fields will have an oscillatory nature. In addition, depending upon the combination of materials, the real part of the stress singularity can have any value between 0 (not singular) and -1 [20]. However, some essential aspects of the delamination problem can be studied using the simple system, consisting of an elastically homogeneous material containing aligned interfaces, discussed here.

2. Splitting

The initiation stage of a split is a complex problem because it depends upon the geometry of the discontinuity that causes delamination. A sharp crack will behave differently from a blunt notch or hole. In the former case there is a singularity in the stress field which may initiate delamination; in the latter case such a singularity develops only after the delamination process has begun. However, it is well recognized that both shear and normal stresses are involved [17, 21].

When the split is more than a few times the length of the notch, the problem becomes much better defined. Eventually, an asymptotic regime is reached in which the stress field at the tip of the delamination is independent of its length. The asymptotic strain-energy release rate can then be readily obtained by considering the changes in elastic-strain energy between the split and unsplit configurations [22]. This can be illustrated for a split originating from a notch of depth c in the side of the specimen (Fig. 2a). Except for a small region near the tip of the split, the material between the split and the free surface is unstressed. The material in the same region was under a uniform tension equal to the applied stress, σ_∞, before

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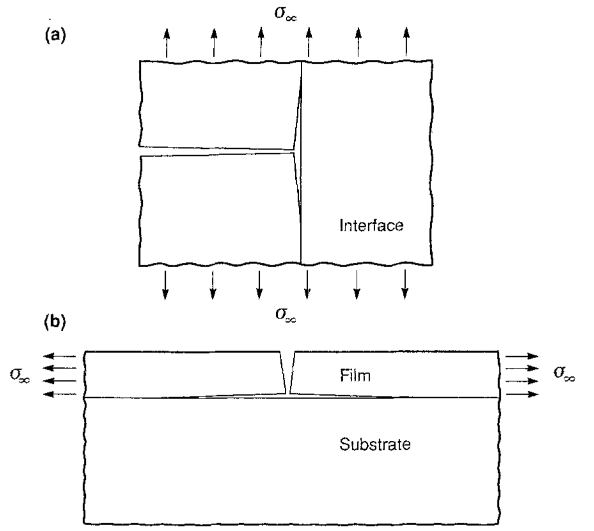

Figure 1 Delamination at interfaces: (a) a composite material under an applied load (b) a thin film under residual tension supported on a substrate.

delamination occurred. The change in the elastic energy of the system (in plane stress) is, therefore [22]

$$U = -\frac{\sigma_\infty^2}{2E} cbl \tag{1}$$

where b is the width of the specimen, l the length of the split and E Young's modulus. Since the applied stress does no work as the delamination advances, the strain energy release rate is defined as

$$\mathscr{G} = -\frac{1}{b}\frac{\partial U}{\partial l} \tag{2}$$

which then yields

$$\mathscr{G} = \frac{c\sigma_\infty^2}{2E} \tag{3}$$

An exactly analogous calculation for a central notch of length $2c$ (Fig. 2b) yields the same value for the asymptotic value for $\mathscr{G}$ if delamination occurs from both ends of the notch.

These simple calculations do not allow the separate mode-I and -II components to be determined. There appears to be a danger that, because of the loading, the problem may be construed to be one of pure mode-II [23]. This is incorrect. Unless it is clear that there is either perfect symmetry or perfect anti-symmetry with respect to a crack tip it must always be assumed that a crack is subject to mixed-mode conditions. The asymptotic values of the stress-intensity factors for the problem of an edge notch can be obtained directly from results derived for the geometry shown in Fig. 3 [24]. This geometry consists of a semi-infinite plane with a sub-surface semi-infinite crack parallel to the free surface at a depth c. There is an applied load of P per unit depth combined with a bending moment of M per unit depth acting on the "arm" of material between the crack and the free surface. The stress-intensity factors are given by [24]

$$K_\text{I} = 0.434Pc^{-1/2} + 1.934Mc^{-3/2} \tag{4a}$$

$$K_\text{II} = 0.558Pc^{-1/2} - 1.503Mc^{-3/2} \tag{4b}$$

Superposition (Fig. 4) shows that the asymptotic stress-intensity factors for the problem of a long split originating from an edge notch in a plate subject to a uniform tension σ_∞ are given by Equation 4 with $M = 0$ and $P = \sigma_\infty c$

$$K_\text{I} = 0.434\sigma_\infty\sqrt{c} \tag{5a}$$

$$K_\text{II} = 0.558\sigma_\infty\sqrt{c} \tag{5b}$$

When the delamination is small in comparison to the length of the notch, stress-intensity factors can be calculated by the boundary-element method [25, 26]. The results for delamination lengths, l, greater than 0.01 of the initial crack length are plotted in Fig. 5. It should be noted that the initial portion of the delamination is stable until it reaches the asymptotic conditions [11]*.

Equation 4 can also be used to determine the asymptotic stress-intensity factors for the delamination that may occur when a beam of depth h which

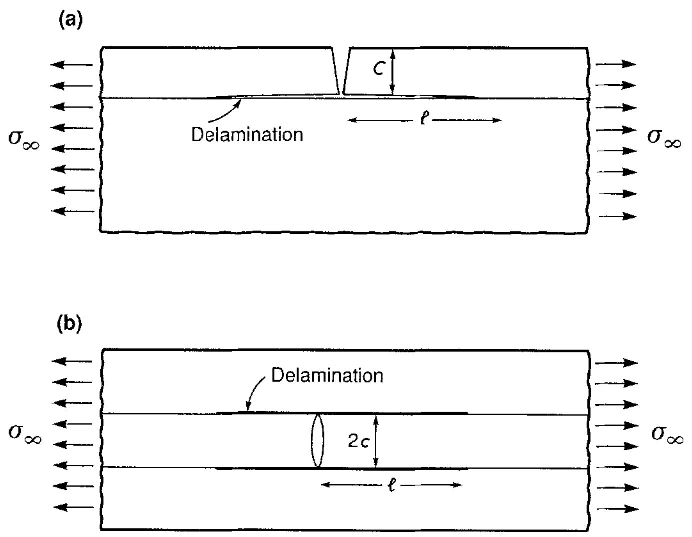

Figure 2 Two important geometries for delamination in composites: (a) from an edge crack, (b) from a central notch.

*The solutions for the problem of a central crack in an infinite plane can be found in a paper by Vitek [27]. It should be noted that K_I is negative once the delamination is more than about one eighth of the original crack length. This suggests that the extensive delamination often observed in a strip of finite width is strongly influenced by the presence of free boundaries.

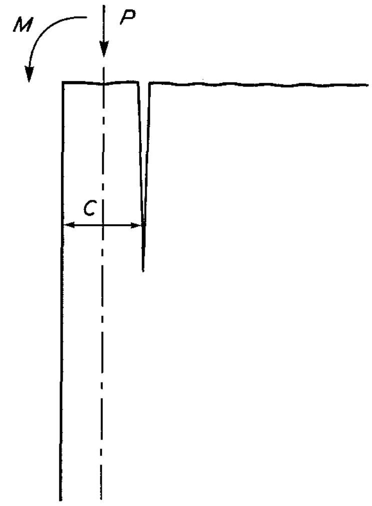

Figure 3 Semi-infinite sub-surface crack with an applied load P and a bending moment M per unit thickness.

contains a shallow edge notch of depth $c (c \ll h)$ is subject to a pure bending moment of T per unit thickness (Fig. 6). This analysis is applicable when delamination is limited to the inner span of a four-point bend specimen. T is then related to the load applied to the specimen, F, and the distance between the inner- and outer-loading points, L

$$T = FL/2 \tag{6}$$

Before delamination occurs, the average load on the "arm" of material between the plane of the split and the free surface is given by simple beam theory as

$$P = \frac{Tc}{h^2}\left(1 - \frac{c}{h}\right) \tag{7}$$

The stress varies linearly across the beam so that the effective bending moment on the "arm" is

$$M = \frac{c^3}{h^3} T \tag{8}$$

Consequently, superposition and the use of Equation 4 yield the asymptotic stress-intensity factors for long delaminations

$$K_I = Th^{-3/2}\left(\frac{c}{h}\right)^{1/2}\left(2.60 - 0.670\frac{c}{h}\right) \tag{9a}$$

$$K_{II} = Th^{-3/2}\left(\frac{c}{h}\right)^{1/2}\left(3.35 - 4.85\frac{c}{h}\right) \tag{9b}$$

provided that $c/h < 0.1$. Numerical calculations are required for geometries in which the notch is deeper. These have been done by Charalambides *et al.* [5] their results suggest the following expressions are valid when $c/h \geqslant 0.2$.

$$K_I = Th^{-3/2}\frac{1}{1 - c/h}\left(0.706 + 3.68\frac{c}{h}\right) \tag{10a}$$

$$K_{II} = Th^{-3/2}\frac{1}{1 - c/h}\left(0.844 + 2.32\frac{c}{h}\right) \tag{10b}$$

3. Discussion

3.1. General concepts

The fracture mechanics of a crack lying in a plane of weakness and subject to a normal tensile stress (Fig. 7a) is very straight forward. If the mode-I stress-intensity factor, K_I, exceeds a critical value for the interface, K_{Ici}, then the crack will extend. An exactly equivalent criterion is that the strain-energy release rate, $\mathcal{G}$, should be greater than or equal to the interface toughness, $\mathcal{G}_{ci}$, where

$$E\mathcal{G}_{ci} = K_{Ici}^2 \tag{11}$$

in plane stress.

The problem is much more complex if a shear stress acts upon the crack. There can then be a mode-II component of the stress-intensity factor at the crack tip (Fig. 7b), and an associated tendency for the crack to extend out of the interface and into the matrix [28]. It appears that K_{II} acts to divert the path of the crack to one for which $K_{II} = 0$ [24, 29–31]. The failure

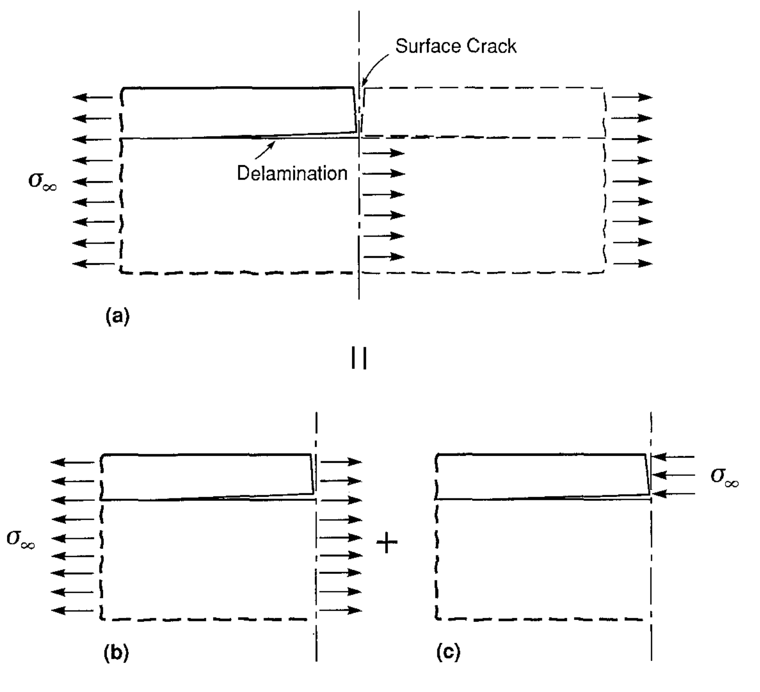

Figure 4 Superposition illustrating how the geometry of Fig. 3 is appropriate for determining the asymptotic stress-intensity factors in the problem of delamination from an edge crack. (Fig. 4b does not contribute to K_I or K_{II}.)

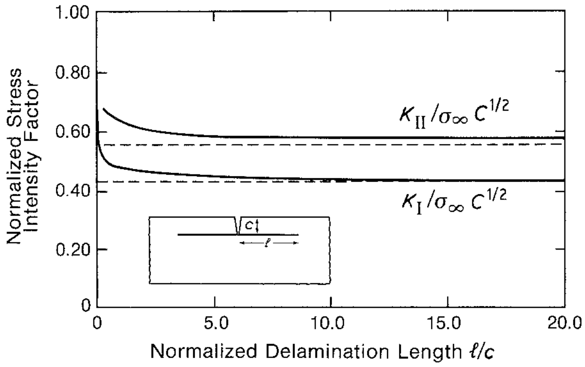

Figure 5 Stress-intensity factors at the tip of a delamination crack in a homogeneous material. (---- asymptotic limit).

criterion could then be a simple one of K_I or $\mathscr{G}$ exceeding critical values (K_{Icm} or $\mathscr{G}_{cm}$ respectively) for the matrix.

If the interface is sufficiently "weak" relative to the matrix, the crack will be confined to the plane of the interface. Under these conditions the trajectory of the crack is prescribed, but the possibility that the mode-II component has some influence on the failure criterion cannot be ignored. There is currently no real understanding of this true mixed mode problem. In practice, the relevant failure criterion appears to lie somewhere between two extremes [32–33]. In one limit only the opening mode contributes to the crack propagation which is assumed to occur when

$$K_I \geqslant K_{Ici} \tag{12a}$$

K_{II} has no influence. The other limit is one in which the critical strain-energy release rate is constant, so that crack propagation occurs when

$$K_I^2 + K_{II}^2 \geqslant E\mathscr{G}_{ci} \tag{12b}$$

A recent study [34] suggests that under mixed-mode loading the critical energy release rate of an interface is a function of K_{II}/K_I ratio, depending on the roughness of the interface.

Whether a crack stays in an interface under mixed-mode loading depends upon the relative "strengths" of the interface and the matrix, and upon the magnitudes of K_I and K_{II}. This is discussed at greater length in the following section. In this discussion a failure criterion must be assumed. In the absence of more certain knowledge, the criterion chosen is that (for the range of K_I/K_{II} pertinent for this problem) the interface and matrix will fail when the strain-energy release rate exceeds $\mathscr{G}_{ci}$ and $\mathscr{G}_{cm}$ respectively. The following analysis can be readily adapted for any other failure criterion.

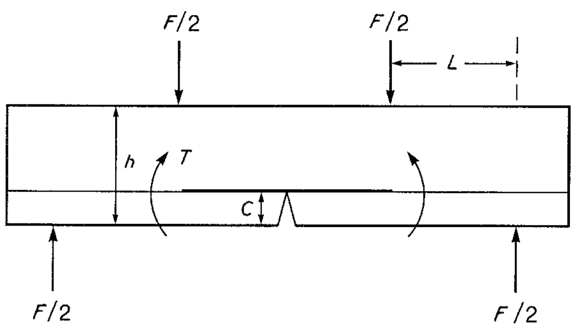

Figure 6 Delamination in a four-point bend specimen, $T = FL/2$.

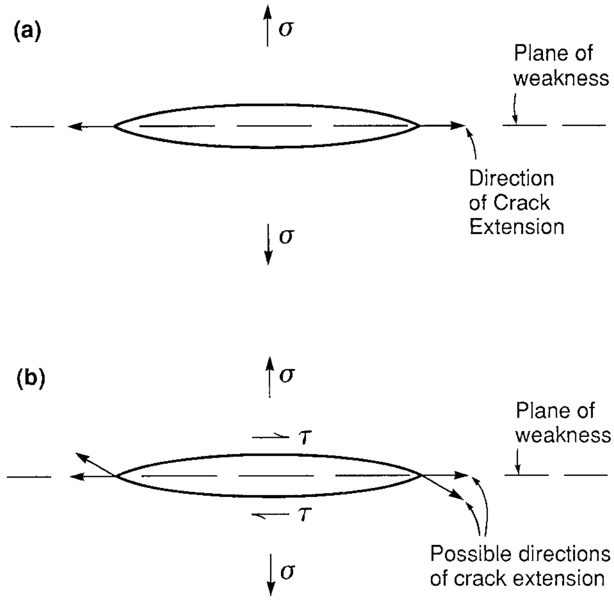

Figure 7 Cracks in interfaces: (a) subject to mode-I loading (b) subject to mixed-mode loading.

3.2. Initial kinking

There are two directions in which a surface crack may propagate when it reaches an interface. Depending upon the stress level and the relative "strengths" of the interface and matrix, the crack may either propagate under pure mode-I loading into the substrate, or it may kink at 90° and grow along the interface (Fig. 2a). The stress-intensity factor at the crack tip is given by [23]

$$\begin{aligned} K_{I_1} &= 1.12\sigma(\pi c)^{1/2} \\ &= 2.0\sigma\sqrt{c} \end{aligned} \tag{13}$$

Consequently, unless

$$\sigma_\infty \geqslant 0.5\left(\frac{E\mathscr{G}_{cm}}{c}\right)^{1/2} \tag{14}$$

the crack *cannot* extend into the matrix.

Approximate conditions under which the crack may kink and grow up the interface can be obtained from the results of Cotterell and Rice [31]. The stress-intensity factors at the tip of an infinitesimal kink inclined at an angle α to the main crack (Fig. 8) are approximately

$$K_I = c_{11}k_1 + c_{12}k_2 \tag{15a}$$

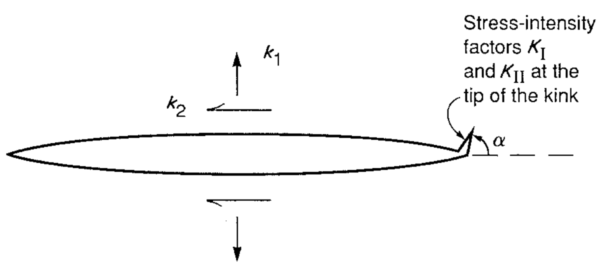

Figure 8 Geometry of a kinked crack.

$$K_{II} = c_{21}k_1 + c_{22}k_2 \quad (15b)$$

where k_1 and k_2 are the stress-intensity factors acting on the main crack and,

$$c_{11} = \tfrac{1}{4}(3\cos\alpha/2 + \cos 3\alpha/2)$$
$$c_{12} = -\tfrac{3}{4}(\sin\alpha/2 + \sin 3\alpha/2)$$
$$c_{21} = \tfrac{1}{4}(\sin\alpha/2 + \sin 3\alpha/2)$$
$$c_{22} = \tfrac{1}{4}(\cos\alpha/2 + 3\cos 3\alpha/2)$$

In the present problem, $\alpha = 90°$, $k_2 = 0$ and k_1 is given by Equation 13. K_I and K_{II} at the tip of an infinitesimal kink at the interface are then given by

$$K_{I_2} = K_{II_2} \approx 0.71\sigma_\infty\sqrt{c} \quad (16)$$

(which is in good agreement with extrapolation from the results of the boundary-element method (Fig. 5)). Therefore, assuming a failure criterion of $\mathcal{G}_c$ = constant, the crack cannot propagate up the interface unless

$$\sigma_\infty \geqslant 1.0\left(\frac{E\mathcal{G}_{ci}}{c}\right)^{1/2} \quad (17)$$

A comparison of Equations 14 and 17 suggests that if

$$\mathcal{G}_{cm} > 4\mathcal{G}_{ci} \quad (18)$$

the stress required for delamination will be reached before the stress required to propagate the crack across the interface. It should be noted that even if this criterion is not satisfied, the crack may be stopped by the interface if local debonding occurs ahead of the crack [1]. Dynamic effects may also be significant in determining the influence of the interface [35].

If tensile tests are conducted to measure the apparent fracture toughness of specimens, different values would be obtained if the notch is oriented along the interface rather than perpendicular to it. Tests in which the notch is aligned with the interface would yield the true value of K_{Ici} (the fracture toughness of the interface). The other tests would yield either K_{Icm}, or a quantity involving the applied stress and notch size required for delamination. If the constant $\mathcal{G}_c$ failure criterion is used, then the magnitude of the quantity $\sigma_\infty\sqrt{c}$ required to initiate delamination will be given by Equation 17. The *apparent* fracture toughness is then

$$K_{ca} = 1.12\sqrt{\pi}\,[1.0(E\mathcal{G}_{ci})^{1/2}] \approx 2\,K_{Ici} \quad (19)$$

In a practical test there might be some ambiguity about the exact location of the precursor crack tip, should it not reach the interface, K_{Icm} would be measured. Alternatively, the ratio of K_{ca}/K_{Icm} would be greater than two if some delamination occurred during the process of introducing the crack. However, reported values for this ratio appear to be even higher, which suggests that other effects such as elastic anisotropy are important [13].

3.3. Cracking the matrix

Experiments on some composite systems [13, 36, 37] (wood [13] and some Al–Li alloys [37] being particular examples) occasionally show failure of the type illustrated schematically in Fig. 9. Delamination occurs from the initial notch, but eventually the split breaks out of the interface and crosses another ply or grain before causing further delamination. The process may be repeated until the crack has crossed the specimen and failure occurs. It is, therefore, of interest to discuss the conditions under which the crack may leave the interface.

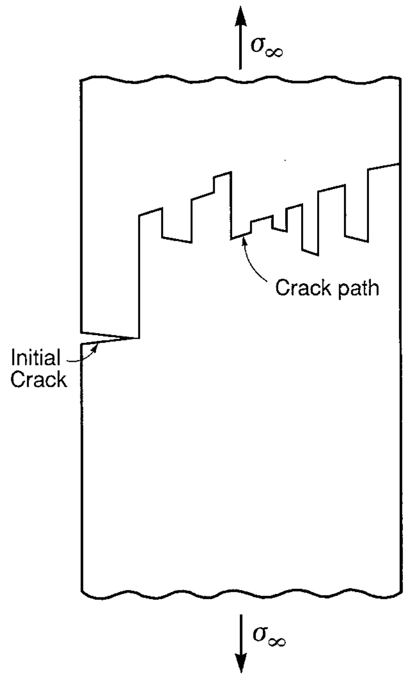

Figure 9 Schematic illustration of the type of failure seen sometimes, for example, in wood [13] where the crack grows along the interfaces and across grains.

When the delamination is long and originates from a shallow edge notch in a tensile specimen (Fig. 2a), the values of k_1 and k_2 in Equation 15 are given by Equation 5. A kink which deflects into the matrix at an angle of approximately 56.6° will have no mode-II component, and the strain-energy release rate will be equal to $0.83\sigma_\infty^2 c/E$. Consequently, for the crack to extend into the matrix (Equations 5 and 15)

$$\sigma_\infty \geqslant 1.1\left(\frac{E\mathcal{G}_{cm}}{c}\right)^{1/2} \quad (20)$$

If alternatively, the split were to continue up the interface, the applied load σ_∞ would have to be such that (Equation 3)

$$\sigma_\infty \geqslant 1.4\left(\frac{E\mathcal{G}_{ci}}{c}\right)^{1/2} \quad (21)$$

Consequently, the crack will deviate into the matrix if

$$\mathcal{G}_{ci} > 0.60\mathcal{G}_{cm} \quad (22)$$

Comparison of Equations 18 and 22 shows that if delamination is induced from a sharp surface crack, an approach based solely on fracture mechanics predicts that the crack will not leave the interface. Since K_I/K_{II} does not change substantially as the interface crack develops (Fig. 5), this interpretation would not be changed by the adoption of anything but

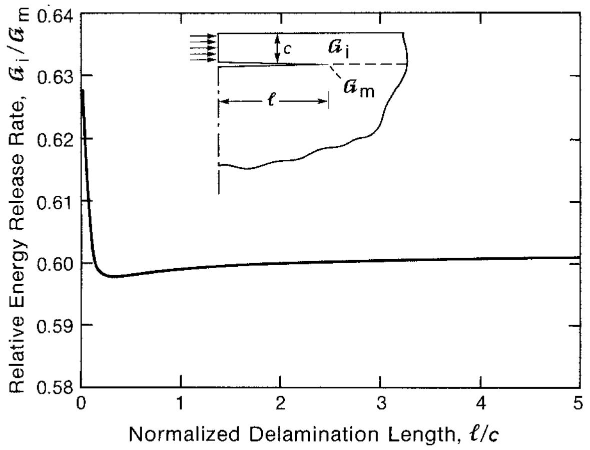

Figure 10 Comparison between the strain-energy release rates of an interface crack and of a crack starting to grow in the matrix at an angle such that $K_{II} = 0$.

a most unlikely failure criterion. Such a criterion would have to entail a rapid change in $\mathcal{G}_c$ over the range $K_I/K_{II} = 1$ to $K_I/K_{II} = 0.78$.

There are conditions under which matrix cracking could follow delamination if, for example, some critical event causes delamination of the interface ahead of the initial crack [1]. Even under these circumstances, Fig. 10 shows that there is a very limited range of the ratio $\mathcal{G}_{ci}/\mathcal{G}_{cm}$ for which delamination would precede matrix cracking.

There is some evidence from recent work on ceramic composites [38] and model systems of alumina bonded with a thin layer of a second material [39, 40] that statistical effects may play an important role. It appears that the matrix sometimes fails from pre-existing flaws that propagate after the delamination process. Delamination may occur at a relatively low load, and the matrix may then fail upon a further increase in load. Alternatively, it appears that in some cases the interface may arrest the initial crack, and then a second crack may propagate on the other side of the interface in response to the shed load [40]. In general, the two cracks would not be co-linear so that the final stage of deformation might involve interaction between them and consequent failure of the interface [40].

4. Conclusions

Some analytical results for delamination caused by geometrical discontinuities at the surface of a semi-infinite plane have been presented. The results are pertinent to problems which are elastically homogeneous but contain a "weak" interface. Conditions for determining whether delamination will occur depend upon the relative toughnesses of the interface and matrix material, as well as the stress levels and geometry. It has been shown that, from a purely fracture mechanics point of view, once delamination has been initiated there will be no tendency for the crack to leave the interface while it remains in a region of constant stress. It is possible, though, that statistical effects could provide an explanation as to why the crack is sometimes observed to deviate from the interface.

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